

Oefeningen algebra ANTWOORDEN

1. $S_{y=-x} : y = 2x^2 - x \rightarrow -x = 2y^2 + y$

2. $S_{x=5} : y = 2\sqrt{x+1} \rightarrow y = 2\sqrt{11-x}$

3. $f(x) = 2x^2 - x$, bereken netjes

$$\begin{aligned} f'(2) &= \left(\frac{df}{dx} \right)_{x=2} = \lim_{\Delta x \rightarrow 0} \frac{f(2+\Delta x) - f(2)}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{2(2+\Delta x)^2 - (2+\Delta x) - 6}{\Delta x} = \\ &= \lim_{\Delta x \rightarrow 0} \frac{8 + 8\Delta x + 2\Delta x^2 - 2 - \Delta x - 6}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{7\Delta x + 2\Delta x^2}{\Delta x} = \lim_{\Delta x \rightarrow 0} 7 + 2\Delta x = 7 \end{aligned}$$

Algebra oefeningen:

4. $(x+2)^3 = x^3 + 6x^2 + 12x + 8$

5. $(x-1)^4 = x^4 - 4x^3 + 6x^2 - 4x + 1$

6. $(2x-1)^3 = 8x^3 - 12x^2 + 6x - 1$

7. $(1-x)^4 = 1 - 4x + 6x^2 - 4x^3 + x^4$

Gebruik regels differentiëren:

8. $f(x) = \sqrt{x} = x^{\frac{1}{2}} \Rightarrow f'(x) = \frac{1}{2} \cdot x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$ $f(x) = \frac{1}{x} = x^{-1} \Rightarrow f'(x) = -1 \cdot x^{-2} = \frac{-1}{x^2}$

$$f(x) = \frac{2}{\sqrt{x}} = 2 \cdot x^{-\frac{1}{2}} \Rightarrow f'(x) = -x^{-\frac{1}{2}} = \frac{-1}{x\sqrt{x}}$$

$$f(x) = \frac{3}{2x} = \frac{3}{2} \cdot \frac{1}{x} \Rightarrow f'(x) = \frac{3}{2} \cdot -1 \cdot x^{-2} = \frac{-3}{2x^2}$$

$$f(x) = \frac{x^2-1}{x} = x - x^{-1} \Rightarrow f'(x) = 1 + x^{-2} = 1 + \frac{1}{x^2}$$

9. $f(x) = \frac{x-3}{\sqrt{x}} = x^{\frac{1}{2}} - 3 \cdot x^{-\frac{1}{2}} \Rightarrow f'(x) = \frac{1}{2} \cdot x^{-\frac{1}{2}} + 1 \frac{1}{2} x^{-\frac{3}{2}} = \frac{1}{2\sqrt{x}} + \frac{3}{2x\sqrt{x}}$

$$f(x) = \frac{2}{3x\sqrt{x}} = \frac{2}{3} \cdot x^{-\frac{3}{2}} \Rightarrow f'(x) = \frac{2}{3} \cdot -\frac{3}{2} \cdot x^{-\frac{5}{2}} = \frac{-1}{x^2\sqrt{x}}$$

$$f(x) = \frac{3x^2}{2\sqrt{x}} = \frac{3}{2} \cdot x^{\frac{3}{2}} \Rightarrow f'(x) = \frac{3}{2} \cdot \frac{3}{2} \cdot x^{\frac{1}{2}} = \frac{9}{4\sqrt{x}}$$

$$f(x) = \frac{5x-2}{3\sqrt{x}} = \frac{5}{3} \cdot x^{\frac{1}{2}} - \frac{2}{3} \cdot x^{-\frac{1}{2}} \Rightarrow f'(x) = \frac{5}{3} \cdot \frac{1}{2} \cdot x^{-\frac{1}{2}} - \frac{2}{3} \cdot -\frac{1}{2} \cdot x^{-\frac{3}{2}} = \frac{5}{6\sqrt{x}} + \frac{1}{3x\sqrt{x}}$$

10 $f(x) = (3x-1)^2 = 9x^2 - 6x + 1 \Rightarrow f'(x) = 18x - 6$

$$f(x) = (2x+1)^3 = 8x^3 + 12x^2 + 6x + 1 \Rightarrow f'(x) = 24x^2 + 24x + 6$$

$$f(x) = (1-2x)^3 = 1 - 6x + 12x^2 - 8x^3 \Rightarrow f'(x) = -6 + 24x - 24x^2$$

Staatdelingen:

Oefenen:

$$\frac{5x-1}{x+1} = 5 + \frac{-6}{x+1}$$

$$\frac{2x+3}{x-1} = 2 + \frac{5}{x-1}$$

$$\frac{6x-7}{2x+1} = 3 + \frac{-10}{2x+1}$$

$$\frac{x^2-6x+8}{x-2} = x-4$$

$$\frac{x^2+3x-10}{x+5} = x-2$$

$$\frac{x^3-8}{x-2} = x^2+2x+4$$

$$\frac{x^3-1}{x-1} = x^2+x+1$$

$$\frac{x^2-12x+48}{x-4} = x-8 + \frac{16}{x-4}$$

$$\frac{5+4x}{x-1} = 4 + \frac{9}{x-1}$$

$$\frac{x^2-1}{x-1} = x+1$$

$$\frac{x^3-8}{x-2} = x^2+2x+4$$

$$\frac{2x^2-10x+12}{x-2} = 2x-6$$

Vergelijking oplossen:

Alle 3 methoden gebruiken:

$$x^2+2x-24=0 \Rightarrow (x+6)(x-4)=0 \Rightarrow x=-6 \vee x=4$$

$$x^2-6x+7=0 \Rightarrow (x-3)^2-2=0 \Rightarrow x=3+\sqrt{2} \vee x=3-\sqrt{2}$$

Ontbinden kan nu ook:

$$x^2-6x+7=0 \Rightarrow (x-3-\sqrt{2})(x-3+\sqrt{2})=0$$

$$x^2-6x+8=0 \Rightarrow (x-4)(x-2)=0 \quad x^2+4x+3=0 \Rightarrow (x+3)(x+1)=0$$

Kies de handigste

$$2x^2-x-6=0 \Rightarrow x^2-\frac{1}{2}x-3=0 \Rightarrow (x-2)(x+1\frac{1}{2})=0$$

$$x^2+x-56=0 \Rightarrow (x-7)(x+8)=0$$

$$x^2+6x-16=0 \Rightarrow (x+3)^2-7=0 \Rightarrow x=-3+\sqrt{7} \vee x=-3-\sqrt{7}$$

$$2x^2-3x-4=0 \Rightarrow x = \frac{3 \mp \sqrt{9+32}}{4}$$

Alles door elkaar:

1. $S_{x=-3} : y = 3\sqrt{x+2} \rightarrow y = 3\sqrt{-4-x}$

2. $S_{y=x} : y = \frac{2x+3}{x-1} \rightarrow x = \frac{2y+3}{y-1} \Leftrightarrow y = \frac{x+3}{x-2}$ (lastig)

3. $(6-x)^3 = 216 - 108x + 18x^2 - x^3$

4. $(x-2)^5 = x^5 - 10x^4 + 40x^3 - 80x^2 + 80x - 32$

5. $f(x) = \frac{2x-1}{3\sqrt{x}} = \frac{2}{3}x^{\frac{1}{2}} - \frac{1}{3}x^{-\frac{1}{2}} \Rightarrow f'(x) = \frac{1}{3}x^{-\frac{1}{2}} + \frac{1}{6}x^{-\frac{1}{2}} = \frac{1}{3\sqrt{x}} + \frac{1}{6x\sqrt{x}}$

6. $f(x) = \frac{5+\sqrt{x}}{3x} = \frac{5}{3}x^{-1} + \frac{1}{3}x^{-\frac{1}{2}} \Rightarrow f'(x) = -\frac{5}{3}x^{-2} - \frac{1}{6}x^{-\frac{1}{2}} = \frac{-5}{3x^2} - \frac{1}{6x\sqrt{x}}$

7. $\frac{5x+4}{2-x} = -5 + \frac{-6}{2-x}$

8. $\frac{3x+2}{x-2} = 3 + \frac{8}{x-2}$

9. $x^2+8x+15=0 \Rightarrow (x+5)(x+3)=0 \Rightarrow x=-5 \vee x=-3$

10. $x^2+6x+7=0 \Rightarrow (x+3)^2-2=0 \Rightarrow x=-3+\sqrt{2} \vee x=-3-\sqrt{2}$