

Differentieer oefening 4 Antwoorden

$$a. \quad f'(x) = \lim_{b \rightarrow 0} \frac{(x+b)^4 - x^4}{b} = \lim_{b \rightarrow 0} 4x^3 + 6x^2b + 4xb^2 + b^3 = 4x^3$$

$$b. \quad f'(x) = \lim_{b \rightarrow 0} \frac{(x+b)^2 - (x+b) - x^2 + x}{b} = \lim_{b \rightarrow 0} \frac{x^2 + 2bx + b^2 - x - b - x^2 + x}{b} \\ = \lim_{b \rightarrow 0} 2x - 1 + b = 2x - 1$$

$$c. \quad f'(x) = \lim_{b \rightarrow 0} \frac{(x+b)^3 - (x+b)^2 - x^3 + x^2}{b} = \\ \lim_{b \rightarrow 0} \frac{x^3 + 3x^2b + 3xb^2 + b^3 - x^2 - 2bx - b^2 - x^3 + x^2}{b} \\ \lim_{b \rightarrow 0} 3x^2 + 3xb + b^2 - 2x - b = 3x^2 - 2x$$

$$d. \quad f'(x) = \lim_{b \rightarrow 0} \frac{\frac{1}{x+b} - \frac{1}{x}}{b} = \lim_{b \rightarrow 0} \frac{x - (x+b)}{x(x+b)b} = \lim_{b \rightarrow 0} \frac{-b}{bx(x+b)} = \frac{-1}{x^2}$$

$$e. \quad f'(1) = \lim_{\Delta x \rightarrow 0} \frac{f(1+\Delta x) - f(1)}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{1 - 2 \cdot (1+\Delta x)^2 - (-1)}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{-4\Delta x - 2\Delta x^2}{\Delta x} = \lim_{\Delta x \rightarrow 0} -4 - 2\Delta x = -4$$

$$a. \quad f(x) = \frac{1}{\sqrt{2x}} \Rightarrow f'(x) = \frac{1}{\sqrt{2}} \cdot -\frac{1}{2} \cdot x^{-\frac{1}{2}}$$

$$b. \quad G = \frac{4X+1}{X} = 4 + X^{-1} \Rightarrow G' = -1 \cdot X^{-2}$$

$$c. \quad Y = \sqrt{1-A} \Rightarrow Y' = \frac{1}{2} \cdot (1-A)^{-\frac{1}{2}} \cdot -1$$

$$d. \quad K = B \cdot \sin B \quad K' = \sin B + B \cdot \cos B$$

$$e. \quad R = \frac{5}{3E} \Rightarrow R' = \frac{5}{3} \cdot -1 \cdot E^{-2}$$

$$f. \quad S = \sqrt{2p(p-3p^2)} = \sqrt{2} \cdot p^{\frac{1}{2}} - 3\sqrt{2} \cdot p^{\frac{3}{2}} \Rightarrow S' = 1 \cdot \frac{1}{2} \cdot \sqrt{2} \cdot p^{-\frac{1}{2}} - 5\sqrt{2} \cdot p^{\frac{1}{2}}$$