

Symmetrie

ANTWOORDEN

1. Toon aan dat de grafiek van $f(x) = x - 2 + \frac{1}{2x - 6}$ symmetrisch is in $(3, 1)$

$$f(3-a) = 3-a-2 + \frac{1}{2(3-a)-6} = 1-a + \frac{1}{-2a} \wedge f(3+a) = 3+a-2 + \frac{1}{2(3+a)-6} = 1+a + \frac{1}{2a} \Rightarrow$$

$$f(3-a) + f(3+a) = 2 \Rightarrow \frac{f(3-a) + f(3+a)}{2} = \frac{2}{2} = 1$$

2. Toon aan dat de grafiek van $f(x) = {}^3\log(1+x^2)$ symmetrisch is in de y-as

$$f(-x) = {}^3\log(1+(-x)^2) = {}^3\log(1+x^2) = f(x)$$

3. Toon aan dat de grafiek van $f(x) = 3x^3 - \frac{2}{x}$ symmetrisch is in $(0, 0)$

$$f(-x) = 3(-x)^3 - \frac{2}{(-x)} = -3x^3 + \frac{2}{x} = -(3x^3 - \frac{2}{x}) = -f(x)$$

4. Toon aan dat de grafiek van $f(x) = \frac{8-2x}{3+x}$ symmetrisch is in $(-3, -2)$

$$\frac{f(-3+a) + f(-3-a)}{2} = \frac{\frac{8-2(-3+a)}{3+(-3+a)} + \frac{8-2(-3-a)}{3+(-3-a)}}{2} =$$

$$\frac{\frac{14-2a}{a} + \frac{14+2a}{-a}}{2} = \frac{1}{2} \cdot \left(\frac{14-2a}{a} + \frac{-14-2a}{a} \right) = \frac{1}{2} \cdot \frac{-4a}{a} = -2$$

5. Toon aan dat de grafiek van $f(x) = \frac{4 \cdot 3^x}{1+9^x}$ symmetrisch is in de y-as

$$f(-x) = \frac{4 \cdot 3^{-x}}{1+9^{-x}} \cdot \frac{9^x}{9^x} = \frac{9^x \cdot 4 \cdot 3^{-x}}{9^x + 1} = \frac{3^{2x} \cdot 4 \cdot 3^{-x}}{9^x + 1} = \frac{4 \cdot 3^x}{1+9^x} = f(x)$$

6. Toon aan dat de grafiek van $f(x) = \frac{5+\cos x}{2\cos x - 3}$ symmetrisch is in de y-as

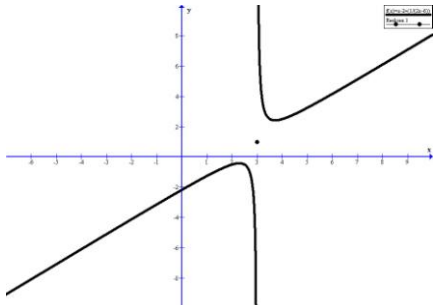
$$f(-x) = \frac{5+\cos(-x)}{2\cos(-x)-3} = \frac{5+\cos x}{2\cos x - 3} = f(x)$$

7. Toon aan dat de grafiek van $f(x) = 4\sin x - 5$ symmetrisch is in $l: x = \frac{1}{2}\pi$

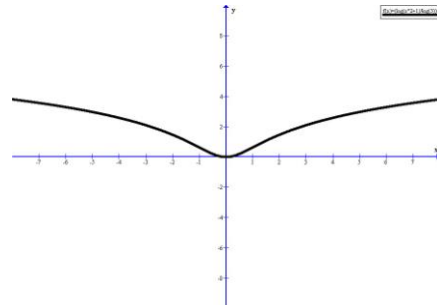
$$f\left(\frac{1}{2}\pi - x\right) = 4\sin\left(\frac{1}{2}\pi - x\right) - 5 = 4\sin\left(\frac{1}{2}\pi + x\right) - 5 = f\left(\frac{1}{2}\pi + x\right)$$

8. Toon aan dat de grafiek van $f(x) = \frac{3}{2x}$ symmetrisch is in $l: y = -x$

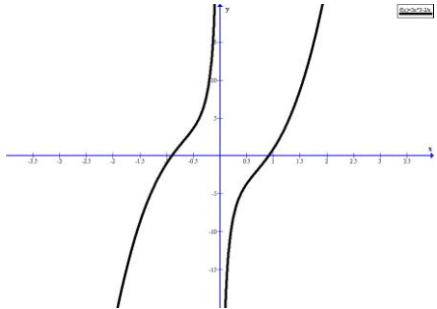
$$f(a) = \frac{3}{2a} = b \Rightarrow f(-b) = f\left(-\frac{3}{2a}\right) = \frac{3}{2 \cdot \left(-\frac{3}{2a}\right)} = -a$$



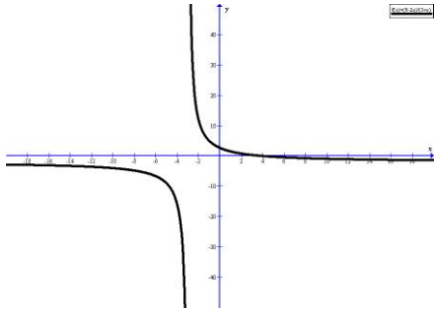
Opgave 1



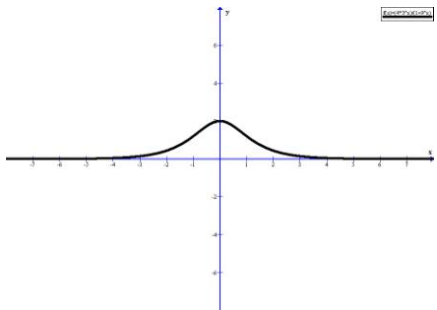
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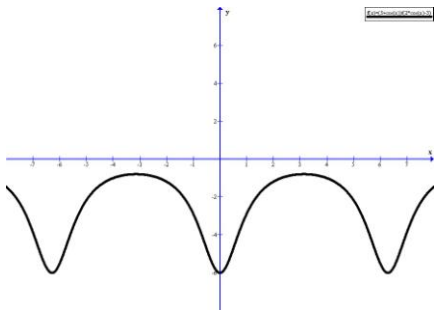
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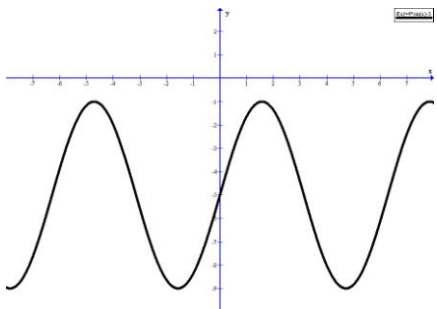
Opgave 4



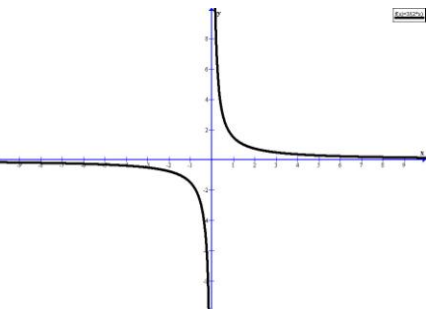
Opgave 5



Opgave 6



Opgave 7



Opgave 8